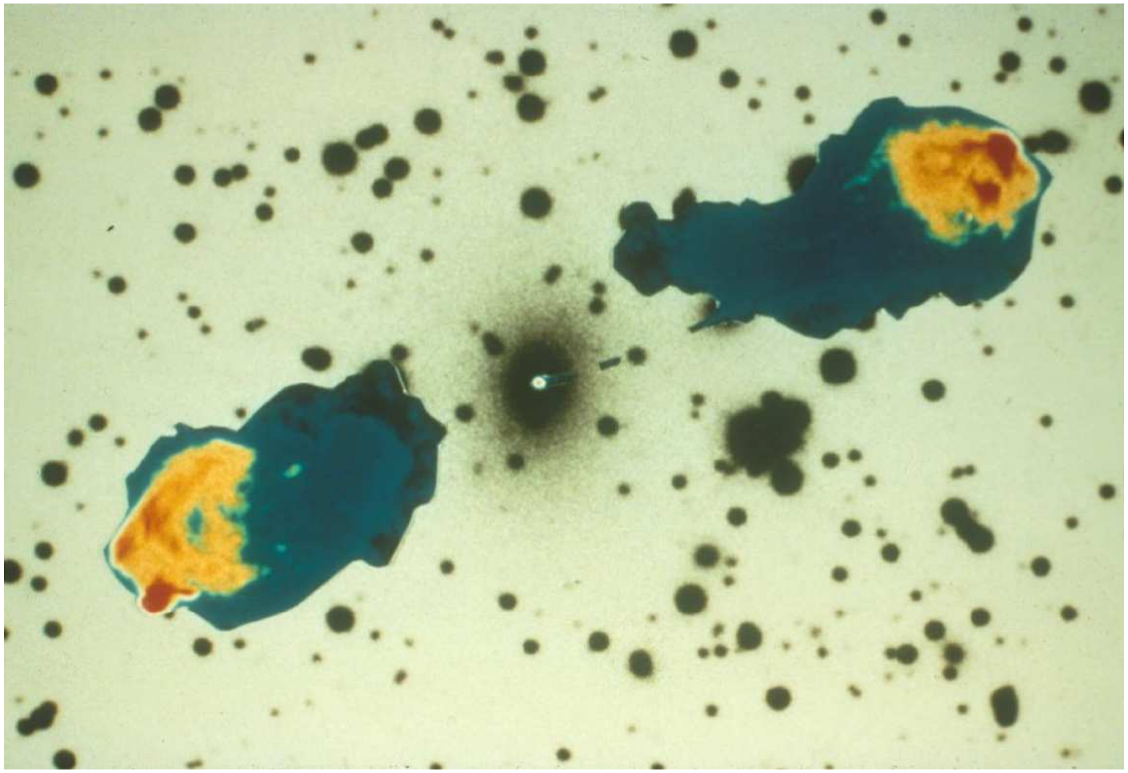
OUTLINE

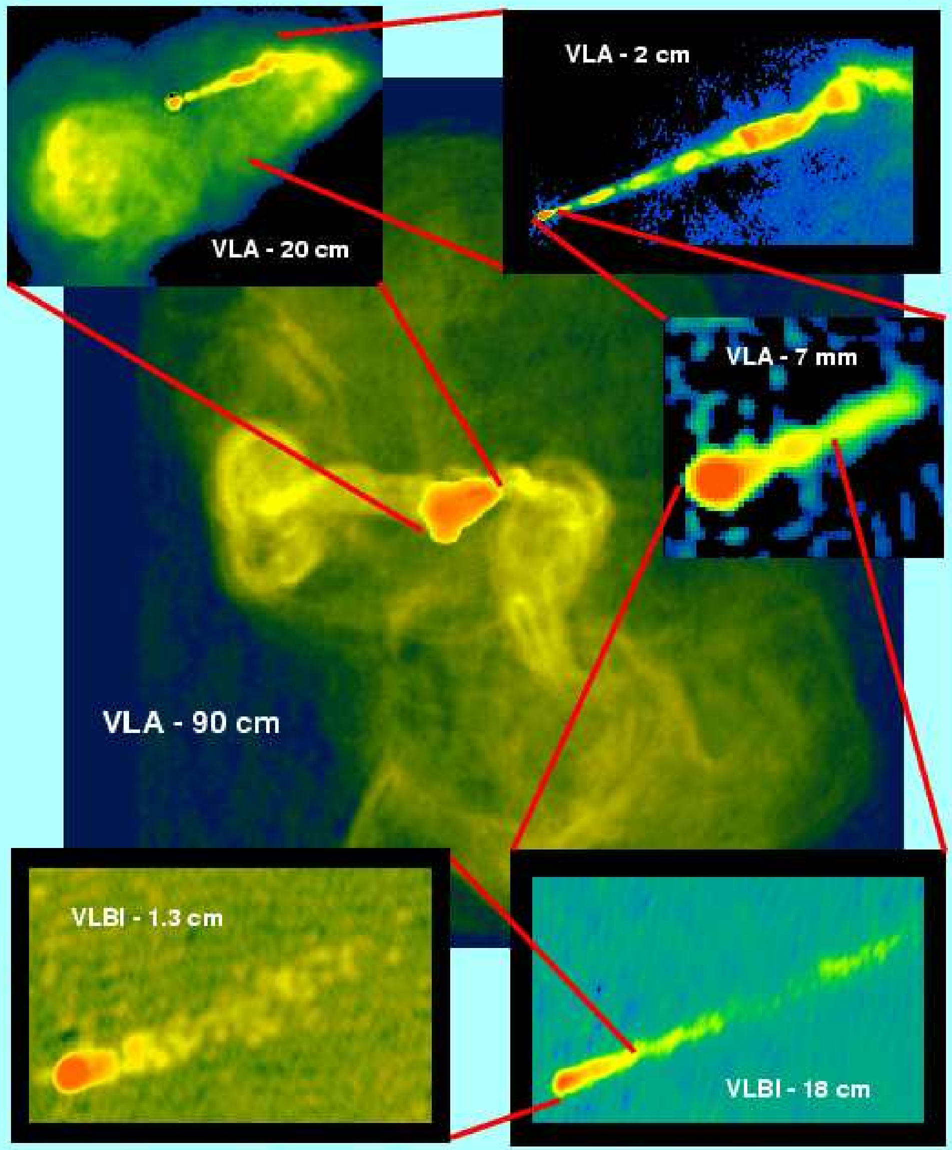
- Astrophysics
- CFD (Jargon)
- Stability Analysis
- Philosophy (Should you believe the results?)

http://www.astro.lsa.umich.edu/users/hughes — Teaching

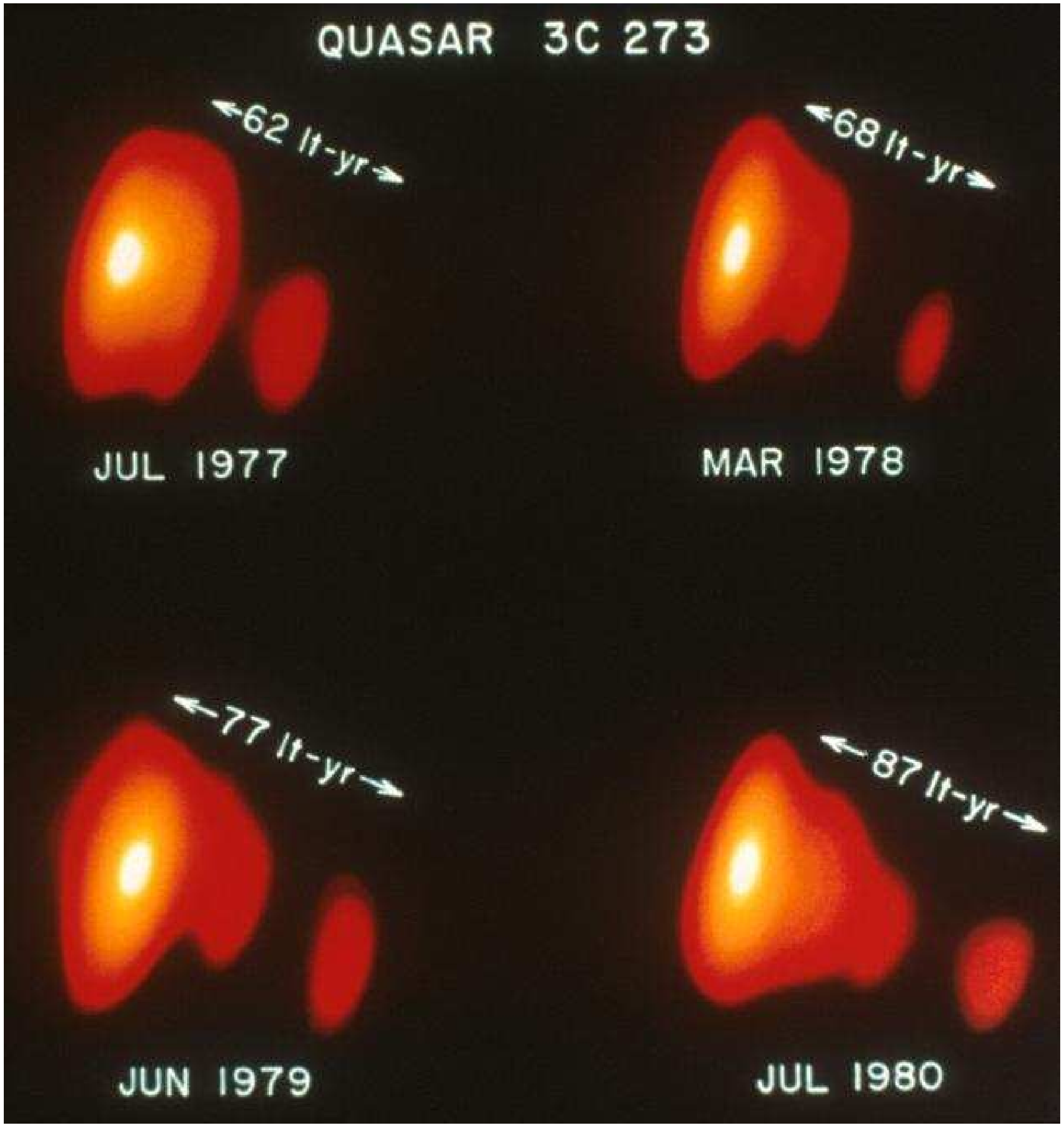
Astrophysics



M87 -- From 200,000 Light-Years to 0.2 Light-Year



Credit: Frazer Owen (NRAO), John Biretta (STScI) and colleagues. The National Radio Astronomy Observatory is a facility of the National Science Foundation, operated under cooperative agreement by Associated Universities, Inc.


QUASAR 3C 273
62 lt-yr
JUL 1977
68 lt-yr
MAR 1978
77 lt-yr
JUN 1979
87 lt-yr
JUL 1980

- $\star$ The objects are non-WYSIWYG !!

- $\star$ We see synchrotron radiation from e^-/e^+ in some B; particle energy distribution is nonthermal; very diffuse.

- $\star$ Flow looks fluid-like because

  - $\triangleright$ $\lambda \equiv 1/n\sigma >> \mathcal{L}$

  but

  - $\triangleright$ $r_L \equiv mv/eB << \mathcal{L}$

  and even for dynamically insignificant B we can have

  - $\triangleright$ $\mathbf{B} \sim \delta\mathbf{B}, \mathcal{L}_B << \mathcal{L}$

- ⋆ Data imply relativistic, collimated "fluid" flow.
- ⋆ Can use CFD, etc. to understand properties of such flows.
- ⋆ But — beware comparison with data.
  - ▷ Radiation is from nonthermal particles, in B *but both are inferred indirectly, not computed!*
  - ▷ Line of sight integration and relativistic effects(*) lead to loss of detail/confusion.

(*)Relativistic effects: boost, time delay ($c = 1$).

$$I_\nu(\nu) = \mathcal{D}^3 I_{\nu'}\left(\nu'\right)$$

$$\mathcal{D} = \frac{1}{\gamma\left(1 - v\cos\theta\right)}$$

$$\gamma = \frac{1}{\left(1 - v^2\right)^{1/2}}$$

CFD

- Express physics as conservation of mass, momentum, energy.
- Discretize spatially (and temporally) into piecewise constant values.
- A 1-D conservation law of form (NB. no sources)

$$U_t + F_U U_x = 0$$

has in volume averaged form

$$\frac{\partial U_i}{\partial t} + \frac{1}{\Delta x}\Sigma_k \mathbf{F}_k \cdot \mathbf{n}_k = 0$$

or

$$\frac{\Delta U_i}{\Delta t} + \frac{1}{\Delta x}\left(F_{i+1/2} - F_{i-1/2}\right) = 0$$

- Update solution by computing fluxes (note physical basis) by solving Riemann problem at each cell interface (exactly or approximately). Robust but inaccurate.
- Gain accuracy by 'reconstructing' cell values on L/R through, e.g. piecewise linear method – "2nd order scheme". Achieve comparable accuracy in time by computing solution/flux at 1/2 time step (predictor-corrector method).
- Some robustness lost; apply a slope limiter to get TVD behavior

$$TV = \Sigma_i |U_{i+1} - U_i|$$

– still 2nd order in smooth regions.

Stability Analysis

Consider a plane, incompressible flow with no gravity, surface tension or viscosity.

Momentum:

$$\rho\left(\partial/\partial t + \mathbf{v}\cdot\nabla\right)\mathbf{v} = -\nabla p$$

Continuity:

$$\partial\rho/\partial t + \mathbf{v}\cdot\nabla\rho = 0$$

Incomp.

$$\nabla\cdot\mathbf{v} = 0$$

Equilibrium (Initial) State:

$$\mathbf{v} = \left(v_0\left(z\right), 0, 0\right)$$
$$\rho = \rho\left(z\right)$$

Perturbations:

$$\mathbf{v} = \left(v_0 + \delta v_x, \delta v_y, \delta v_z\right)$$
$$\rho = \rho_0 + \delta\rho$$
$$p = p_0 + \delta p$$

Linearize:

$$\rho_0 \frac{\partial \delta v_x}{\partial t} + \rho_0 v_0 \frac{\partial \delta v_x}{\partial x} + \rho_0 \delta v_z \frac{\partial v_0}{\partial z} = -\frac{\partial \delta p}{\partial x}$$

ETC.

Normal Modes Analysis:

$$\delta Q = \delta Q_0 \exp\left[i\omega t + ik_x x + ik_y y\right]$$

NB. No z!!

Leads to

$$\rho_0 i \left(\omega + v_0 k_x\right) \delta v_x + \rho_0 \delta v_z \partial v_0 / \partial z + ik_x \delta p = 0$$

ETC.

Eliminate:

$$D \equiv \partial/\partial z$$

$$D\left(\rho_0\left(\omega + v_0 k_x\right) D\delta v_z\right) - D\left(k_x \rho_0 \delta v_z D v_0\right) - k^2 \rho_0 \left(\omega + v_0 k_x\right)\delta v_z = 0$$

Specialize to vortex sheet:

$$v_0 = v_0^{[1]},\ z > z_s$$
$$v_0 = v_0^{[2]},\ z < z_s$$

similarly for ρ. Will let $z_s = 0$.

For either region (1) or (2)

$$\left(D^2 - k^2\right)\delta v_z = 0$$

Thus

$$\delta v_z = C_1 \exp\left(-kz\right) + C_2 \exp\left(+kz\right)$$

where evidently

$$C_1 = 0,\ z < 0;\ \ C_2 = 0,\ z > 0$$

Choice of constants is a bit subtle!

$$\delta v_z|_{z=z_s} = \frac{d}{dt}\delta z|_{z=z_s}$$

or, linearizing

$$\delta v_z|_{z=z_s} = \frac{\partial}{\partial t}\delta z_s + v_0|_s \frac{\partial}{\partial x}\delta z_s$$

normal modes:

$$\delta v_z|_{z=z_s} = i\left(\omega + k_x v_0|_s\right)\delta z_s$$

whence

$$\delta z_s \propto \frac{\delta v_z|_{z=z_s}}{(\omega + k_x v_0|_s)}$$

LHS is continuous at interface, so we must have

$$\delta v_z|_{z=z_s} \propto (\omega + k_x v_0|_s)$$

so must choose

$$C_{1,2} = C\left(\omega + k_x v_0^{[1,2]}\right)$$

Match at boundary

$$\lim_{\epsilon\to 0}\int_{z_S-\epsilon}^{z_S+\epsilon} \mathrm{LHS}\, dz = 0$$

TYPE-1 terms:

$$\lim_{\epsilon\to 0}\int \left[\frac{d}{dz}Q\right] dz \Rightarrow \Delta_s Q$$

where $\Delta_s Q \equiv Q|_{z_S+0} - Q|_{z_S-0}$.

TYPE-2 terms:

$$\lim_{\epsilon\to 0}\int Q dz \Rightarrow 0$$

Thus, finally,

$$\Delta_s\{\rho_0\left(\omega + k_x v_0\right) D\delta v_z - k_x \rho_0 \delta v_z D v_0\} = 0$$

$D\delta v_z$ is known, $Dv_0 = 0$.

Putting it all together:

$$\alpha_{1,2} \equiv \frac{\rho_0^{[1,2]}}{\rho_0^{[1]} + \rho_0^{[2]}}$$

We get a DISPERSION RELATION

$$\omega = -k_x \left(\alpha_1 v_0^{[1]} + \alpha_2 v_0^{[2]}\right) \pm \left(-k_x^2 \alpha_1 \alpha_2 \left(v_0^{[1]} - v_0^{[2]}\right)^2\right)^{1/2}$$

Given some k_x

$$\delta Q \propto \exp\left[i\omega t\right] \propto \exp\left[-\Im\left(\omega\right) t\right] \exp\left[i\Re\left(\omega\right) t\right]$$

First term determines *growth time scale*, for mode of *frequency* in second term.

For a perturbed jet, we would set ω, find k and

$$\Im\left(k\right) \Rightarrow \text{Growth Length}$$

$$\Re\left(k\right) \Rightarrow \text{Wavelength of mode}$$

There will be integer number of nodes around jet circumference, pressure nulls on radius.

Philosophy

Never fly in a plane designed by an astronomer.

Numerical schemes that experience suggests are "robust" get used with few checks, and applied in unexplored parts of parameter space.

We could do better:

<u>(Local) von Neumann Stability Analysis</u>

$$Q\left(x\right)^{n} = G^{n} Q_{0} \exp\left[ikx\right]$$

$$\frac{Q^{n+1}}{Q^{n}} = \frac{G^{n+1}}{G^{n}} \equiv G$$

Is

$$|G| < 1$$

for some $\Delta x, \Delta t$? Need to check at each place/time, but could at least subsample.

Given evidence of stability, use

Lax Theorem

$$\|Q^T - Q^n\| \propto \Delta x^\alpha$$

Compute for multiple Δx, thus estimate "error" in numerical solution.

But usually can barely get result for one Δx of useful resolution, in available cycles.

Trend is to get sanity checks from melding simulation/analysis/data into self-consistent picture.

That also covers possibility of quite spurious physics from, e.g., wrong microphysics in

$$Re \equiv \frac{UL}{\nu} \qquad Rm \equiv \frac{UL}{\eta}$$